Solutions to Practice Quiz 6

Problem 1a)

The minimum current will be such that the torques are balanced. The torque on a loop immersed in a uniform B is:

$\tau_1 = \vec{\mu} \times \vec{B}$ where $\vec{\mu} = NI\vec{A}$ where A is an area vector $\boxed{A = 2RW}$ & N the number of loops

The torque due to the mass M is:

$\tau_2 = \vec{r} \times \vec{F} = \vec{r} \times m\vec{g}$

where $\vec{r}$ is the lever-arm: the distance between the axis of rotation and where the force is applied

Let the angle between $\vec{\mu}$ & $\vec{B}$ be α and the angle between $\vec{r}$ and $\vec{g}$ be β. then.

$\tau_1 = \tau_2$

$\mu B \sin\alpha = Rmg\sin\beta$

$NI2WB\sin\alpha = mg\sin\beta$. However, $\alpha = \pi - \beta$

$NI2WB\sin(\pi - \beta) = mg\sin(\beta)$. $\sin(\pi - \theta) = \sin(\theta)$

$$\boxed{I = \frac{mg}{N2WB}}$$

Problem 5b.

Here, the best way to solve this is by conservation of Energy.

The work done by the magnetic torque equals the energy gained by the box

note that while θ goes from $0 \to \pi$, β goes from $\pi \to 0$

$$W = \int_0^{\pi} \tau \, d\theta = -\int_{\pi}^{0} \mu B \sin\beta \, d\beta = -\mu B(\cos(\pi) - \cos(0)) = 2\mu B$$

$$W_{ext} = U = mgh_{top} = 2 \times 2RWNIB$$

$$h_{top} = \frac{4RWIBN}{mg} \quad . \quad \text{if } \sigma = \frac{2NIWB}{mg}, \text{ then}$$

$$\boxed{h_{top} = 2\sigma R}$$

Problem 2. a) As the particle goes up, the magnetic force turns it to the right. Eventually the magnetic force and the Electric force cancel out; then the motion starts again.

Problem 2b. A constant E is like a gravitational field $\vec{g}$ where $g = E$, the particle potential energy qEy

$E_{mech} = -Eqy + \frac{1}{2}mv^2 = 0$ the kinetic energy is due to a potential loss

$$v = \sqrt{\frac{2qEy}{m}}$$

Problem 2c $\quad qE - qvB = -\frac{v^2}{2y}m = -qE$

$$\boxed{v = \frac{2E}{B}}$$

Problem 3

$$q(\vec{E} + \vec{v} \times \vec{B}) = m\vec{a} \qquad \vec{E} = E\hat{j}$$
$$\vec{B} = B\hat{k}$$

$$qE\hat{j} + v_x B(\hat{i} \times \hat{k}) = ma_y$$

$$v_y B(\hat{j} \times \hat{k}) = ma_x$$

$$\boxed{\begin{aligned} qE + v_y B &= m\frac{dv_y}{dt} \\ v_y B &= m\frac{dv_x}{dt} \end{aligned}}$$