Solution to practice quiz 5

$$x = Ae^{i\omega t} + Be^{-i\omega t}$$

$$\left.\begin{aligned} x(0) &= A + B = 10 \\ \dot{x}(0) &= i\omega A - i\omega B = 2 \\ A - B &= \frac{2}{i\omega} \end{aligned}\right\}$$

$$2A = 10 + \frac{2}{i\omega}$$

$$\boxed{\begin{aligned} A &= \frac{10}{2} - i\frac{1}{\omega} \\ B &= A^* = \frac{10}{2} + i\frac{1}{\omega} \end{aligned}}$$

$$\omega = \sqrt{\frac{k}{m}} = 2$$

$$\text{Amplitude} = 2|A| = 2\sqrt{5^2 + \frac{1}{4}} \sim 10.05 \text{ m} \quad \checkmark$$

You could also do it with trig identities

$$\begin{aligned} x &= A\cos(\omega t + \phi) \\ \dot{x} &= -A\omega \sin(\omega t + \phi) \end{aligned} \Rightarrow \begin{aligned} x(0) &= A\cos(\phi) = 10 \\ \dot{x}(0) &= -A\omega\sin(\phi) = 2 \end{aligned}$$

$$\phi = \cos^{-1}\left(\frac{10}{A}\right) ?$$

Harder to solve.

Problem 2.

$$\text{Energy} = \frac{1}{2}kA^2 = \frac{1}{2}kAe^{-\frac{\gamma}{2m}t}$$

$$E(0) = \frac{1}{2}kA$$

$$\frac{E(t)}{E(0)} = e^{-\frac{\gamma}{2m}t} = \sqrt{0.5} \Rightarrow \text{key eqn.}$$

$$\ln(\sqrt{0.5}) = -\frac{\gamma}{2m}t$$

$$\frac{2m}{\gamma}\ln(2) = t \Rightarrow t = 4\ln(2)$$

Problem 3. This one is trivial. You do it.

Problem 4. The CM of the sphere will advance as usual

$$E = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2 \quad ; \quad r\omega = v_{cm}$$

$$\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\frac{v_{cm}^2}{r^2} = \frac{1}{2}kx^2 \quad ; \quad I = \frac{1}{2}Mr^2$$

$$\frac{1}{2}Mv_{cm}^2 + \frac{1}{4}Mv_{cm}^2 = \frac{1}{2}kx^2$$

$$v_{cm} = \sqrt{\frac{2}{3}\frac{k}{M}}x \Rightarrow$$ then with this you set the two energies