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The Problem of Coordination

A scientific theory is typically presented with laws, principles, or equations that involve terms specific to that theory:

$$s = vt + (1/2)at^2, PV = rT, F = ma, \text{action} = \text{reaction}$$

to mention only a few of the classical standbys. Although these terms are, taken literally, symbols for functions with specified mathematical character, they are often pronounced as if they were nouns already familiar before the theory's introduction: "distance", "velocity", "time", "pressure", "temperature", and so forth. That nomenclature is introduced more by way of informal commentary than explanation, and certainly does not define the theoretical terms.¹ But the choice of familiar words does signify something: they point to the sort of data to enter, and the sorts of measurements that can help to determine the values of those functions. The theory would remain a piece of pure mathematics, and not an empirical theory at all, if its terms were not linked to measurement procedures. But what is this linkage?

That question, which turns out to bring many further questions and complexities in its train, poses what was once generally known as the '*problem of coordination*'.

(...)

The questions *What counts as a measurement of (physical quantity) X?* and *What is (that physical quantity) X?* cannot be answered independently of each other. To echo another such realization, I am not ashamed to admit that this brings us to the famed 'hermeneutic circle' (Eco 1992: 64). We shall examine this apparent circularity by focusing on the one hand on its more abstract consideration by Reichenbach, and on the other hand the practical response in history examined by Mach and Poincaré

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In 1920 Reichenbach, wants to find a general coordination of mathematical spaces and their structure with physical relations. In his later *Philosophy of Space and Time* he gives as first example how units of length can receive their coordination: “a meter is the forty-millionth part of the circumference of the earth” (Reichenbach 1958: 15). But the recurring example of the concept of a straight line, or more generally a geodesic, as having as physical correlate a light ray, or the path of a freely falling body, is not equally easy to understand. Nor is the idea that congruence relations have as correlate coincidence with transported rigid bodies.⁵ In these examples it is quite unclear how to identify the physical correlate without using any geometric or kinematic terms. How are we to describe rigidity or free

fall without using the language of geometry, or of mathematical physics in general? In the example that most preoccupied him about the theory of relativity, non-simultaneity is to be thus related to non-connectability through any signals, whether by light emission and reflection or by material transport. Not only the modal character of “connectable” but also the required identity-over-time (genidentity) of the material are as puzzlingly theoretical as any of the terms in physical geometry. And so we find him perplexed. He writes in 1920:⁶

It is characteristic of modern *physics* to represent all processes in terms of *mathematical* equations. But the close connection between the two sciences must not blur their essential difference. (Reichenbach 1965: 34)

The mathematical object of knowledge is uniquely determined by the axioms and definitions of mathematics. (*Ibid.*)

The *physical object* cannot be determined by axioms and definitions. It is a thing of the real world, not an object of the logical world of mathematics. Offhand it looks as if the method of representing physical events by mathematical equations is the same as that of mathematics. Physics has developed the method of defining one magnitude in terms of others by relating them to more and more general magnitudes and by ultimately arriving at “axioms”, that is, the fundamental equations of physics. Yet what is obtained in this fashion is just a system of mathematical relations. What is lacking in such system is a statement regarding the significance of physics, the assertion that the system of equations is *true for reality*. (*Ibid.*: 36)

So how can empirical significance be achieved? The examples he has in mind, as we just saw, are the use of rigid bodies as choice to set the relation of spatial congruence (in effect, to measure length), the choice of a light ray path in vacuo as *physical correlate* for geodesics, the choice of a certain periodic process as setting the unit of time. Question: how are these physical correlates to be identified without use of geometric or kinematic terms?

Is the choice in question simply a choice of a *function*? A function relating what to what? Isn’t a function a mathematical object itself, defined in terms of a relation between *mathematical* objects? So Reichenbach writes:

The coördination performed in a physical proposition is very peculiar. It differs distinctly from other kinds of coördination. For example, if two sets of points are given, we establish a correspondence between them by coördinating to every point of one set a point of the other set. For this purpose, *the elements of each set must be defined*; that is, for each element there must exist another definition in addition to that which determines the coördination to the other set. Such definitions are lacking on one side of the coördination dealing with the cognition of reality. Although the equations, that is, the conceptual side of the coördination, are uniquely defined, the “real” is not. On the contrary, the “real” is defined by coördination to the equations. (Ibid.: 37–8; my italics)⁷

Here Reichenbach was imagining, and discounting, the following naïve sort of reply:

what is called for is simply a function, a mapping, between mathematical objects and physical objects or processes—what is puzzling about that?

The reason he discounts it is because to define a function we need to have the domain and range identified first—and the question at issue was precisely how that can be done without presupposing that we already have a physical-mathematical relation on hand.

So what does Reichenbach mean, when he seems to point to a solution with the words “On the contrary, the “real” is defined by coordination to the equations”? (...) To effect the necessary coordination of abstract mathematical structure to concrete empirical reality he posited a special class of mathematic-al-physical principles—“coordinating principles” or “axioms of coordination”—whose role is precisely to insure those conditions of possibility. These principles, Reichenbach argues at that point, are to be taken as given or imposed a priori, and so to be sharply distinguished from mere empirical laws (“axioms of connection”). (...) Reichenbach can coordinate a mathematical representation with physical objects, events, and processes only in a context where something is already given that will make that possible.