Astronomy AST1002 Name: ______________________________

Gravity Problems (50 pts)

Introduction

You will soon see that gravity plays a rather large role in this course. We will use the equation you learned this week to describe the surface gravity of a world, tidal forces between worlds, resonances, formation of satellites and rings, and tidal heating of planets in the Solar System. We'll start today with the straightforward calculation of the surface gravity of a world and how changing basic properties, like the world's mass and radius, influence the surface gravity of the world.

Goals

This worksheet is designed to give you some practice solving the types of gravity problems you are likely to encounter throughout the first half of this course. Make sure you learn how to solve these problems in a breeze.

Procedure

Newton's Law of Universal Gravitation, or the gravitational force of a world, is represented by the following equation:

$$Fg = \frac{GMm}{R^2}$$

For the purposes of this worksheet, Fg represents the surface gravity of the world in Newtons (N). M is the mass of the world in kilograms (kg) and m is the mass of the object on the surface of the world in kilograms (kg). R is the radius of the world in meters (m). G is the gravitational constant, approximately equal to 6.67×10^{-11} Nm^2kg^{-2}.

I will explain the above formula in some detail, and, more importantly, we will discuss the simplified version of this equation for use in the following problems:

$$Fg \propto \frac{M}{R^2}$$

Questions

1. (2pts) If you were to triple the size of the Earth ($R = 3R_{\oplus}$) and double the mass of the Earth ($M = 2M_{\oplus}$), how much would it change the gravity on the Earth ($Fg = XFg_{\oplus}$)?

2. (2pts) If you decrease the size of the Earth by a half ($R = 1/2R_{\oplus}$) and double the mass of the Earth ($M = 2M_{\oplus}$), how much would it change the gravity on the Earth ($Fg = XFg_{\oplus}$)?

3. (2pts) You have discovered a planet that is one quarter the radius of Earth ($R_p = 1/4R_{\oplus}$) and one half as massive ($M_p = 1/2M_{\oplus}$). How does the gravity on the surface of this planet compare to the gravity on the surface of Earth ($Fg_p = XFg_{\oplus}$)?

4. (2pts) You have discovered a planet that has the same gravity as the Earth ($Fg_p = 1Fg_\oplus$), but is only 1/3 as massive ($M_p = 1/3M_\oplus$). How would the size of this planet compare to the size of the Earth ($R_p = XR_\oplus$)?

5. (2pts) You have discovered a planet that has twice the gravity of Earth ($Fg_p = 2Fg_\oplus$), but is only 1/2 the size of Earth ($R_p = 1/2R_\oplus$). How would the mass of the planet compare with the mass of Earth ($M_p = XM_\oplus$)?

6. (2pts) The gravity of the Moon is about 1/6 that of the Earth. For example, an astronaut weighing 180 pounds on the Earth would weigh about 30 pounds on the Moon. If you were to double the distance between the Earth and the Moon, how much would our 180 pound astronaut weigh on the Moon? Explain your answer.

Below are a few more mystery planet problems for practice. For each problem, solve for the unknown value (X) of your mystery planet. Make sure to show all your work including units.

7. (2pts) Planet Mass = $4M_{\oplus}$ Planet Radius = $2R_{\oplus}$ Planet Gravity = $XFg_{\oplus}$

8. (2pts) Planet Mass = $XM_{\oplus}$ Planet Radius = $4R_{\oplus}$ Planet Gravity = $2Fg_{\oplus}$

9. (2pts) Planet Mass = $1/9M_{\oplus}$ Planet Radius = $XR_{\oplus}$ Planet Gravity = $1Fg_{\oplus}$

10. (2pts) Planet Mass = $1/3M_{\oplus}$ Planet Radius = $2/3R_{\oplus}$ Planet Gravity = $X\ Fg_{\oplus}$

Atmospheric Escape

The ability of a planet to hold onto an atmosphere depends predominantly on two factors: ***Temperature*** and ***Gravity***. The temperature of a planet is important because it is really just a measure of how fast, on average, the molecules of gas in the atmosphere are moving. The higher the temperature, the faster the molecules are moving. The gravity of a planet is important because it determines the escape velocity of a planet. Any object with a velocity greater than the escape velocity will escape the gravitational pull of the planet.

Goals

You will use the following data and equations to determine whether the given worlds will hold particular elements or molecules.

The gravity of a planet is determined by its mass and radius. A planet with a stronger gravitational pull will have a higher escape velocity. Table 1 lists the escape velocities and distances for a few worlds in our solar system.

The temperature of a planet is determined mainly by its distance from the Sun. Table 2 shows the temperature a planet would have at various distances from the Sun.

Planet	**V_{esc} (m/s)**	**Distance (AU)**
Jupiter	59,500	5.2
Earth	11,200	1.0
Mars	5,000	1.5
Moon	2,300	1.0
Ceres	510	2.8

Table 1

Dist. (AU)	0.5	1	2	4	6	8
Temp. (K)	566	400	283	200	163	141

Table 2

The speed of a molecule of gas in an atmosphere depends on its temperature and on its mass. A heavier molecule moves slower than a light molecule at the same temperature. The mass of a molecule is measured in atomic mass units (amu). On the back of this page is a periodic table. The mass of each element is given as the number at the bottom of each box. To find the mass of a molecule, just add up the masses of each element. A few molecules and their masses are given in Table 3 on the next page.

The speed of a molecule of gas (in m/s) can be determined from the equation:

$$Vgas = 157\sqrt{\frac{Temperature}{molecular\ mass}}$$

A “rule of thumb” in planetary science is that a planet can hold onto a gas **for the age of the solar system** if the velocity of the gas is less than one sixth the escape velocity of the planet:

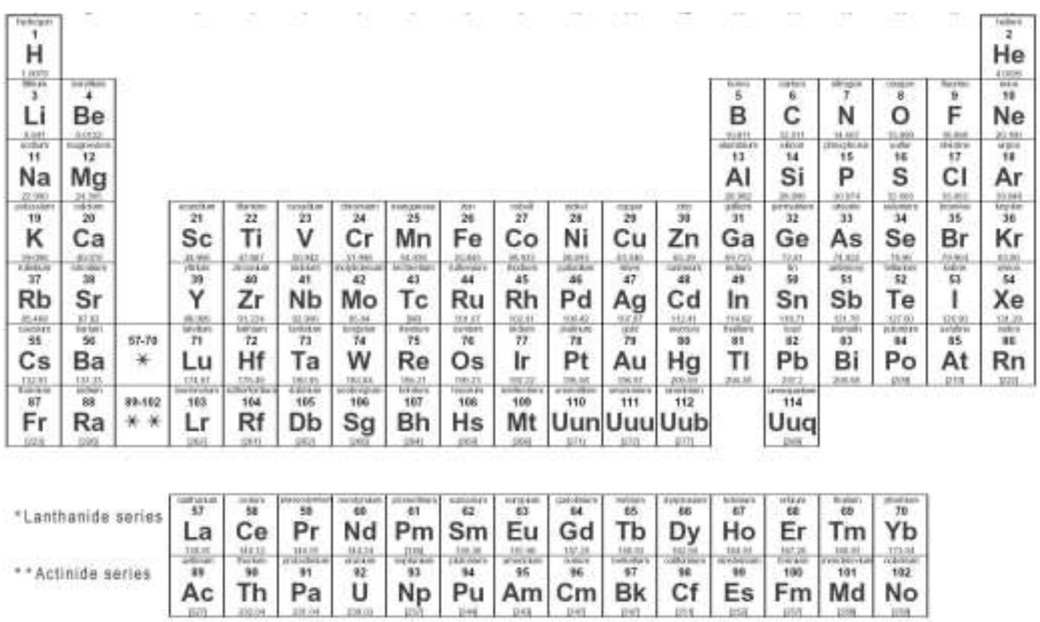
$$Vgas < \frac{1}{6}\,Vesc$$

Name	Symbol	Mass
Hydrogen	H_2	2
Methane	CH_4	16
Ammonia	NH_3	17
Water	H_2O	18
Nitrogen	N_2	28
Carbon Dioxide	CO_2	44

Table 3

For example, the escape velocity of earth is 11,200 m/s. 1/6 X 11,200 m/s = 1, 867 m/s, so the Earth can hold onto any gas with a speed less than 1,867 m/s.

Periodic Table of Elements



Procedure

Using the tables and equations given on the previous pages and a calculator, compute the requested values, showing all work (including appropriate units). Explain your final results and conclusions in a sentence or two for each problem.

Questions

1. (2pts) What is the velocity of a hydrogen molecule at 1 AU?

2. (4pts) Can the Earth hold onto an atmosphere of hydrogen? Explain why or why not.

3. (2pts) What is the velocity of a nitrogen molecule at 1 AU?

4. (4pts) Can the Earth hold onto an atmosphere of Nitrogen? Explain why or why not.

5. (4pts) Estimate **how far** the Moon would have to be from the Sun before it would be cool enough to retain a nitrogen atmosphere.

For the following questions, use the equations given on the previous pages to solve for the missing variable, as you did with question 5.

6. (4pts) What gasses from Table 3 could Mars hold onto?

7. (4pts) What gasses from Table 3 could Ceres hold onto?

8. (4pts) Jupiter formed at currently lives around 5 AU from the Sun. Could Jupiter hold onto an atmosphere of hydrogen molecules if you moved it to 0.5 AU from the Sun?

9. (2pts) Given what you currently know about the formation of the planets in the Solar System, could Jupiter have ***formed*** at 0.5 AU from the Sun? Explain your answer.