

Introduction

You will soon see that gravity plays a rather large role in this course. We will use the equation you learned this week to describe the surface gravity of a world, tidal forces between worlds, resonances, formation of satellites and rings, and tidal heating of planets in the Solar System. We'll start today with the straightforward calculation of the surface gravity of a world and how changing basic properties, like the world's mass and radius, influence the surface gravity of the world.

Goals

This worksheet is designed to give you some practice solving the types of gravity problems you are likely to encounter throughout the first half of this course. Make sure you learn how to solve these problems in a breeze.

Procedure

Newton's Law of Universal Gravitation, or the gravitational force of a world, is represented by the following equation:

$$F_g = \frac{GMm}{R^2}$$

For the purposes of this worksheet, F_g represents the surface gravity of the world in Newtons (N). M is the mass of the world in kilograms (kg) and m is the mass of the object on the surface of the world in kilograms (kg). R is the radius of the world in meters (m). G is the gravitational constant, approximately equal to $6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$.

I will explain the above formula in some detail, and, more importantly, we will discuss the simplified version of this equation for use in the following problems:

$$F_g \propto \frac{M}{R^2}$$

Questions

1. (2pts) If you were to triple the size of the Earth ($R = 3R_{\oplus}$) and double the mass of the Earth ($M = 2M_{\oplus}$), how much would it change the gravity on the Earth ($F_g = XF_g_{\oplus}$)?

2. (2pts) If you decrease the size of the Earth by a half ($R = 1/2R_{\oplus}$) and double the mass of the Earth ($M = 2M_{\oplus}$), how much would it change the gravity on the Earth ($F_g = XF_g_{\oplus}$)?

3. (2pts) You have discovered a planet that is one quarter the radius of Earth ($R_p = 1/4R_{\oplus}$) and one half as massive ($M_p = 1/2M_{\oplus}$). How does the gravity on the surface of this planet compare to the gravity on the surface of Earth ($F_{g_p} = XF_{g_{\oplus}}$)?

4. (2pts) You have discovered a planet that has the same gravity as the Earth ($F_{g_p} = 1F_{g_{\oplus}}$), but is only $1/3$ as massive ($M_p = 1/3M_{\oplus}$). How would the size of this planet compare to the size of the Earth ($R_p = XR_{\oplus}$)?
5. (2pts) You have discovered a planet that has twice the gravity of Earth ($F_{g_p} = 2F_{g_{\oplus}}$), but is only $1/2$ the size of Earth ($R_p = 1/2R_{\oplus}$). How would the mass of the planet compare with the mass of Earth ($M_p = XM_{\oplus}$)?
6. (2pts) The gravity of the Moon is about $1/6$ that of the Earth. For example, an astronaut weighing 180 pounds on the Earth would weigh about 30 pounds on the Moon. If you were to double the distance between the Earth and the Moon, how much would our 180 pound astronaut weigh on the Moon? Explain your answer.

Below are a few more mystery planet problems for practice. For each problem, solve for the unknown value (X) of your mystery planet. Make sure to show all your work including units.

7. (2pts) Planet Mass = $4M_{\oplus}$ Planet Radius = $2R_{\oplus}$ Planet Gravity = XFg_{\oplus}

8. (2pts) Planet Mass = XM_{\oplus} Planet Radius = $4R_{\oplus}$ Planet Gravity = $2Fg_{\oplus}$

9. (2pts) Planet Mass = $1/9M_{\oplus}$ Planet Radius = XR_{\oplus} Planet Gravity = $1Fg_{\oplus}$

10. (2pts) Planet Mass = $1/3M_{\oplus}$ Planet Radius = $2/3R_{\oplus}$ Planet Gravity = $X Fg_{\oplus}$

Atmospheric Escape

The ability of a planet to hold onto an atmosphere depends predominantly on two factors: **Temperature** and **Gravity**. The temperature of a planet is important because it is really just a measure of how fast, on average, the molecules of gas in the atmosphere are moving. The higher the temperature, the faster the molecules are moving. The gravity of a planet is important because it determines the escape velocity of a planet. Any object with a velocity greater than the escape velocity will escape the gravitational pull of the planet.

Goals

You will use the following data and equations to determine whether the given worlds will hold particular elements or molecules.

The gravity of a planet is determined by its mass and radius. A planet with a stronger gravitational pull will have a higher escape velocity. Table 1 lists the escape velocities and distances for a few worlds in our solar system.

Planet	V _{esc} (m/s)	Distance (AU)
Jupiter	59,500	5.2
Earth	11,200	1.0
Mars	5,000	1.5
Moon	2,300	1.0
Ceres	510	2.8

Table 1

Dist. (AU)	0.5	1	2	4	6	8
Temp. (K)	566	400	283	200	163	141

Table 2

The speed of a molecule of gas in an atmosphere depends on its temperature and on its mass. A heavier molecule moves slower than a light molecule at the same temperature. The mass of a molecule is measured in atomic mass units (amu). On the back of this page is a periodic table. The mass of each element is given as the number at the bottom of each box. To find the mass of a molecule, just add up the masses of each element. A few molecules and their masses are given in Table 3 on the next page.

The speed of a molecule of gas (in m/s) can be determined from the equation:

$$V_{\text{gas}} = 157 \sqrt{\frac{\text{Temperature}}{\text{molecular mass}}}$$

A “rule of thumb” in planetary science is that a planet can hold onto a gas **for the age of the solar system** if the velocity of the gas is less than one sixth the escape velocity of the planet:

$$V_{\text{gas}} < \frac{1}{6} V_{\text{esc}}$$

For example, the escape velocity of earth is 11,200 m/s. $1/6 \times 11,200 \text{ m/s} = 1,867 \text{ m/s}$, so the Earth can hold onto any gas with a speed less than 1,867 m/s.

Name	Symbol	Mass
Hydrogen	H ₂	2
Methane	CH ₄	16
Ammonia	NH ₃	17
Water	H ₂ O	18
Nitrogen	N ₂	28
Carbon Dioxide	CO ₂	44

Table 3

Periodic Table of Elements

Hydrogen 1 H 1.00794																		Helium 2 He 4.002602																	
Lithium 3 Li 6.941		Beryllium 4 Be 9.012182																Boron 5 B 10.811		Carbon 6 C 12.011		Nitrogen 7 N 14.0064		Oxygen 8 O 15.9994		Fluorine 9 F 18.9984		Neon 10 Ne 20.1797							
Sodium 11 Na 22.98976928		Magnesium 12 Mg 24.304																Aluminum 13 Al 26.9815386		Silicon 14 Si 28.0855		Phosphorus 15 P 30.973762		Sulfur 16 S 32.06		Chlorine 17 Cl 35.453		Argon 18 Ar 39.948							
Potassium 19 K 39.0983		Calcium 20 Ca 40.078		Scandium 21 Sc 44.955912		Titanium 22 Ti 47.88		Vanadium 23 V 50.9415		Chromium 24 Cr 51.9961		Manganese 25 Mn 54.938045		Iron 26 Fe 55.845		Cobalt 27 Co 58.933195		Nickel 28 Ni 58.6934		Copper 29 Cu 63.546		Zinc 30 Zn 65.38		Gallium 31 Ga 69.723		Germanium 32 Ge 72.630		Arsenic 33 As 74.9216		Selenium 34 Se 78.96		Bromine 35 Br 79.904		Krypton 36 Kr 83.80	
Rubidium 37 Rb 85.4678		Strontium 38 Sr 87.62		Yttrium 39 Y 88.90584		Zirconium 40 Zr 91.224		Niobium 41 Nb 92.90638		Molybdenum 42 Mo 95.94		Technetium 43 Tc [98]		Ruthenium 44 Ru 101.07		Rhodium 45 Rh 102.9055		Palladium 46 Pd 106.42		Silver 47 Ag 107.8682		Cadmium 48 Cd 112.411		Indium 49 In 114.818		Tin 50 Sn 118.710		Antimony 51 Sb 121.757		Tellurium 52 Te 127.6		Iodine 53 I 126.905		Xenon 54 Xe 131.29	
Cesium 55 Cs 132.90545196		Barium 56 Ba 137.327		Lanthanum 57 La 138.90547		Cerium 58 Ce 140.12		Praseodymium 59 Pr 140.90766		Neodymium 60 Nd 144.242		Promethium 61 Pm [145]		Samarium 62 Sm 150.36		Europium 63 Eu 151.964		Gadolinium 64 Gd 157.25		Terbium 65 Tb 158.92535		Dysprosium 66 Dy 162.50014		Holmium 67 Ho 164.93033		Erbium 68 Er 167.259		Thulium 69 Tm 168.93032		Ytterbium 70 Yb 173.05468		Lutetium 71 Lu 174.967		Hafnium 72 Hf 178.49	
Francium 87 Fr [223]		Radium 88 Ra [226]		Actinium 89 Ac [227]		Thorium 90 Th 232.0377		Protactinium 91 Pa 231.036888		Uranium 92 U 238.02891		Neptunium 93 Np [237]		Plutonium 94 Pu [244]		Americium 95 Am [243]		Curium 96 Cm [247]		Berkelium 97 Bk [247]		Californium 98 Cf [251]		Einsteinium 99 Es [252]		Fermium 100 Fm [257]		Mendelevium 101 Md [258]		Nobelium 102 No [259]		Lawrencium 103 Lr [262]			
Ununennium 111 Uue [289]		Unbinilium 112 Uub [289]		Untrium 113 Uut [289]		Unquadrium 114 Uuq [289]		Unpentium 115 Uup [289]		Unsextium 116 Uus [289]		Unseptium 117 Uus [289]		Unoctium 118 Uuo [289]		Unnonium 119 Uun [289]		Undecium 120 Uud [289]		Undwium 121 Udw [289]		Untrivium 122 Uut [289]		Unquadrium 123 Uuq [289]		Unpentium 124 Uup [289]		Unsextium 125 Uus [289]		Unseptium 126 Uus [289]		Unoctium 127 Uuo [289]		Unnonium 128 Uun [289]	

Questions

1. (2pts) What is the velocity of a hydrogen molecule at 1 AU?
2. (4pts) Can the Earth hold onto an atmosphere of hydrogen? Explain why or why not.
3. (2pts) What is the velocity of a nitrogen molecule at 1 AU?
4. (4pts) Can the Earth hold onto an atmosphere of Nitrogen? Explain why or why not.

5. (4pts) Estimate **how far** the Moon would have to be from the Sun before it would be cool enough to retain a nitrogen atmosphere.

For the following questions, use the equations given on the previous pages to solve for the missing variable, as you did with question 5.

6. (4pts) What gasses from Table 3 could Mars hold onto?
7. (4pts) What gasses from Table 3 could Ceres hold onto?
8. (4pts) Jupiter formed at currently lives around 5 AU from the Sun. Could Jupiter hold onto an atmosphere of hydrogen molecules if you moved it to 0.5 AU from the Sun?
9. (2pts) Given what you currently know about the formation of the planets in the Solar System, could Jupiter have **formed** at 0.5 AU from the Sun? Explain your answer.