

Name \_\_\_\_\_

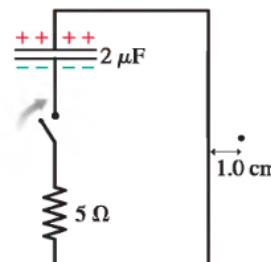
## PHY2049C, Homework 6

**A- Submit a handwritten version of the solutions (clearly readable) at the beginning of class.**

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### Problem 1

The capacitor in the figure is charged to 50 V. The switch closes at  $t = 0$  s. Draw a graph showing the magnetic field strength as a function of time at the position of the dot. On your graph indicate the maximum field strength, and provide an appropriate numerical scale on the horizontal axis.



### Problem 2

When a rigid charge distribution with charge  $Q$  and mass  $M$  rotates about an axis, its magnetic moment  $\mu$  is linearly proportional to its angular momentum  $L$ , with  $\mu = \alpha L$ . The constant of proportionality  $\alpha$  is called the gyromagnetic ratio of the object. We can write  $\alpha = g(Q/2M)$ , where  $g$  is a dimensionless number called the  $g$ -factor of the object. Consider a spherical shell with mass  $M$  and uniformly distributed charge  $Q$  centered on the origin  $O$  and rotating about the  $z$ -axis with angular speed  $\omega$ .

(a) A thin slice with latitude  $\theta$  measured with respect to the positive  $z$ -axis describes a current loop with width  $R d\theta$  and radius  $r = R \sin \theta$  as shown in Figure 1. What is the differential current  $dI$  carried by this loop in terms of  $Q$ ,  $\omega$ ,  $R$ ,  $\theta$  and  $d\theta$ ,

(b) The differential magnetic moment contributed by that slice is  $d\mu = A dI$ , where  $A = \pi r^2$  is the area enclosed by the loop. Express the differential magnetic moment in terms of  $Q$ ,  $\omega$ ,  $R$ ,  $\theta$  and  $d\theta$ . What is the differential current  $dI$

(c) Integrate over  $\theta$  to determine the magnetic moment  $\mu$

(d) what is the magnitude of the angular momentum  $L$ ?

(e) Determine the gyromagnetic ratio  $\alpha$ .

(f) What is the  $g$ -factor for a spherical shell?

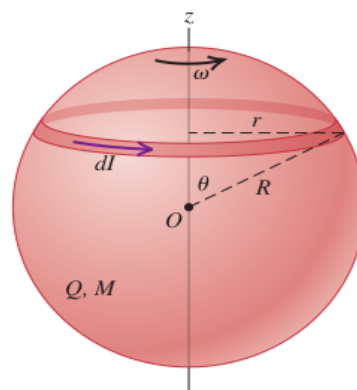


Figure 1

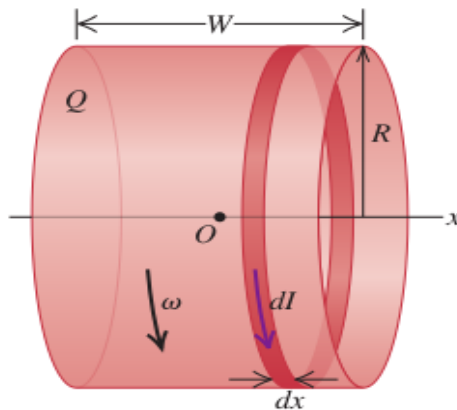
### Problem 3

A cylindrical shell with radius  $R$  and length  $W$  carries a uniform charge  $Q$  and rotates about its axis with angular speed  $\omega$ . The center of the cylinder lies at the origin  $O$  and its axis is coincident with the  $x$ -axis, as shown the Figure below

- (a) What is the charge density  $\sigma$ ?
- (b) What is the differential current  $dI$  on a circular strip of the cylinder centered at  $x$  and with width  $dx$ ?
- (c) Use Eq. (12.16) (see below) to write an expression for the differential magnetic field  $d\vec{B}$  at the origin due to this strip. Note that on the figure, the coordinate  $x$  corresponds to the coordinate  $y$  in the equation.

$$\vec{B} = \frac{\mu_0 \mu \hat{j}}{2\pi(y^2 + R^2)^{3/2}}.$$

- (d) Integrate to determine the magnetic field at the origin.



#### Problem 4

A rectangular loop of length  $l$  and width  $w$  is located a distance  $a$  from a long, straight wire, as shown in Figure 2. What is the mutual inductance of this arrangement?

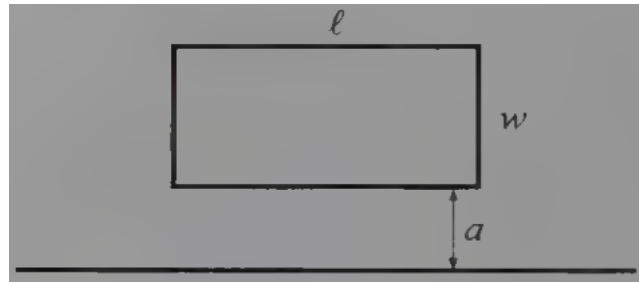


Figure 2

#### Problem 5 (Wolfson and Passachof)

A wire of length  $l$  and resistance  $R$  is formed into a closed rectangular loop twice as long as it is wide. It is mounted on a nonconducting horizontal axle parallel to its longer dimension, as shown in Figure 3. A uniform magnetic field  $\mathbf{B}$  points into the page, as shown. A long string of negligible mass is wrapped many times around a drum of radius  $a$  attached to the axle, and a mass  $m$  is attached to the string. When the mass is released it falls and eventually reaches a speed that, averaged over one cycle of the loop's rotation, is constant from one rotation to the next. Find an expression for that average terminal speed.

